

classicpres

A GAP Package

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1

The classicpres package

This package provides functionality for presentations of classical groups as given in [LGO20]. It is a translation (with permission of the authors) of the corresponding code in the system Magma, and should – short of arbitrary choices depending on internal ordering of finite field elements – return the same data.

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1.1 Functions Provided

There are only two user functions:

1 ► `ClassicalStandardPresentation(type, d, q)`

Given the specification *type*, *d*, *q* of a quasisimple group *G*, this operation constructs a presentation on the standard generators for *G*. The string *type* must be one of "SL", "Sp", "SU", "Omega", "Omega-", or "Omega+", while *d* is the dimension and *q* is the cardinality of the finite field. The presentations are described in [LGO20]. The presentation is returned as a finitely presented group, the relations being stored as its `RelatorsOfFpGroup`

If the option `Projective` is set to true, the operation constructs a presentation for the corresponding projective group (on the images of the same generators).

```
gap> pres:=ClassicalStandardPresentation( "SL" , 4, 3);
<fp group on the generators [ f1, f2, f3, f4 ]>
gap> RelatorsOfFpGroup(pres);
[ f4*f1^-1*f3*f1*f3^2*f1^-1*f3*f1*f3^-1,
  f3^-1*f2^-6*f1^-1*f2^6*f3*f2^-6*f1*f2^6,
  f3^-1*f2^-3*f3^-1*f2^3*f3*f2^-3*f3*f1^-1*f2^3*f3^-1*f2^-3*f1*f2^3,
  f3^-1*f2^-3*f1^-1*f2^3*f3^-1*f2^-3*f1*f2^3*f3*f2^-3*f1^-1*f2^3*f3*f2^-3*f1*f\
2^3, f3^-1*f2^-3*f1^-1*f3^-1*f1*f2^3*f3*f2^-3*f1^-1*f3*f1*f2^3,
  f3^-1*f2^-6*f3^-1*f2^6*f3*f2^-6*f3*f2^6, f1^4,
  f1^-1*f2^-3*f1^2*f2^3*f1^3*f2^-3*f1^2*f2^3, (f1*f2^-3*f1*f2^3)^3,
  f1^-1*f2^-6*f1^-1*f2^6*f1*f2^-6*f1*f2^6, f2^9*(f1^-1*f2^-3)^2*f1^-1,
  f2^-12*f1^-1*f2^12*f1, f2^24, f1*(f3*f1^-1)^2*f3,
  (f1^-1*f3^4*f1^-1*f3^2)^2*f3^3*f1^-2, f2^8 ]
gap> s:=Subgroup(pres, [pres.1,pres.2,pres.4]);
Group([ f1, f2, f4 ])
gap> Index(pres,s);
63180
gap> DisplayCompositionSeries(Image(FactorCosetAction(pres,s)));
G (4 gens, size 6065280)
  A(3,3) = L(4,3) ~ D(3,3) = O+(6,3)
  1 (0 gens, size 1)
```

2 ▶ `ClassicalStandardGenerators(type, d, q)`

This operation produces the standard generators of Leedham-Green and O'Brien for the quasisimple classical group of specified type in dimension d over a field of size q . The type is designated by the argument *type* which must be one of the strings "SL", "Sp", "SU", "Omega", "Omega-", or "Omega+". The standard generators generate a specific copy of a classical group and are defined in [LGO09] and [DLGLO13].

```
gap> gens:=ClassicalStandardGenerators( "SL" , 4, 3);;
[ < immutable compressed matrix 4x4 over GF(3) >,
  < immutable compressed matrix 4x4 over GF(3) >,
  < immutable compressed matrix 4x4 over GF(3) >,
  < immutable compressed matrix 6x4 over GF(3) > ]
gap> List(gens,Order);
[ 4, 8, 3, 2 ]
gap> Size(Group(gens));
12130560
gap> List(RelatorsOfFpGroup(pres),x->Order(MappedWord(x,
> FreeGeneratorsOfFpGroup(pres),gens)));
[ 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1 ]
```

If the option `Projective` is set to true, the operation constructs a presentation for the corresponding projective group (on the images of the same generators).

```
gap> pres:=ClassicalStandardPresentation( "SL" , 4, 3:Projective);
<fp group on the generators [ f1, f2, f3, f4 ]>
gap> List(RelatorsOfFpGroup(pres),x->Order(MappedWord(x,
> FreeGeneratorsOfFpGroup(pres),gens)));
[ 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1 ]
```

1.2 Isomorphism to finitely presented group

In GAP, the canonical way of obtaining a presentation of (say) a permutation group is the operation `IsomorphismFpGroup`. This package thus also installs a method for `IsomorphismFpGroup` for groups that know that they are simple, and tests whether the groups are classical. If so, an isomorphism between the given group, and the permutation action of the classical groups on vectors is computed (this should be improved with constructive recognition in the future) and the presentation of `ClassicalStandardPresentation` is used.

Non-simple groups that construct a presentation through their composition factors will then automatically use these presentations for the simple factors when combining to a presentation of the group.

However be aware that the isomorphisms returned will use the generic permutation group mechanism for decomposition, and thus will not result in good or short words.

1.3 Standard Generators

The generating sequences chosen (and on which the presentations are written) use the standard generators of classical groups as defined in [LGO09] and [DLGLO13], since these generators also are used in the constructive recognition process.

The presentations in [LGO20] initially use slightly different generating sets, and a translation between these two generating sets is provided. If one of the two functions `ClassicalStandardGenerators` or `ClassicalStandardPresentation` is called with the option `PresentationGenerators`, this different generating set is used. (Its use could be to obtain a slightly shorter presentation in situations in which adherence to the standard generators is not required.)

The method provided for `IsomorphismFpGroup` currently uses these `PresentationGenerators`, but this is not a guaranteed property.

Bibliography

- [DLGLO13] Heiko Dietrich, C. R. Leedham-Green, Frank Lübeck, and E. A. O'Brien. Constructive recognition of classical groups in even characteristic. *J. Algebra*, 391:227–255, 2013.
- [LGO09] C. R. Leedham-Green and E. A. O'Brien. Constructive recognition of classical groups in odd characteristic. *J. Algebra*, 322(3):833–881, 2009.
- [LGO20] C. R. Leedham-Green and E. A. O'Brien. Presentations on standard generators for classical groups. *J. Algebra*, 545:324–389, 2020.

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This index covers only this manual. A page number in *italics* refers to a whole section which is devoted to the indexed subject. Keywords are sorted with case and spaces ignored, e.g., “PermutationCharacter” comes before “permutation group”.

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